CALCULATION OF THE TEMPERATURE DISTRIBUTION ON A POROUS PLATE

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On the basis of the relations obtained in [6, 7], a graph is proposed which makes it easy to calculate the temperature distribution on a porous wall with uniform injection of a homogeneous gas. The calculation is compared with available experimental data.

The numerous investigations of heat transfer on permeable surfaces [1–5] have dealt, in the main, with average heat transfer. It was shown in [6] that, to attain the condition T_W = const along a porous plate, the flow rate of injected gas must vary as $\overline{X}^{-0.2}$. In

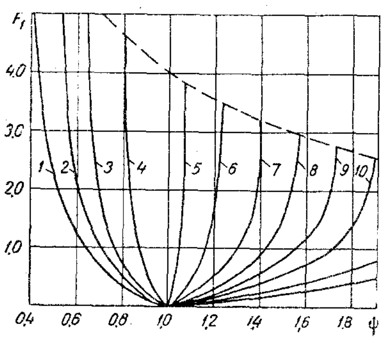

Fig. 1. Dependence of F_1 on ψ for calculating the temperature of a porous wall with $d = T^1/T_0$: 1) 0.392; 2) 0.529; 3) 0.640; 4) 0.812; 5) 1.066; 6) 1.23; 7) 1.40; 8) 1.56; 9) 1.73; 10) 1.89 (the dotted line is the critical injection limit).

the case of uniform injection, then, and this is the regime most often realized in practice, the wall temperature will vary along the length. The calculation of the distribution $T_W = f(\overline{X})$ may be carried out using the energy equation, the law of heat transfer, and the boundary relations for the permeable surface [6]. For these conditions the energy equation, allowing for a velocity gradient, will be [7]

$$\frac{d\,\mathrm{Re}_T^{**}}{d\overline{X}} + \frac{\mathrm{Re}_T^{**}}{\Delta T}\frac{d(\Delta T)}{d\overline{X}} = \mathrm{Re}_0\tilde{U}\mathrm{St}_0(\Psi_s + b_T), \quad (1)$$

where

$$\Psi_s = Kb_T, \quad (2)$$

$$K = (T_W - T')/(T_0 - T_W), \quad (3)$$

$$b_T = \bar{j}_W/\tilde{U}\,\mathrm{St}_0. \quad (4)$$

Performing simple transformations, we have

$$d(\Delta\,\mathrm{Re}_T^{**}) = \mathrm{Re}_0\bar{j}_W(T_0 - T')\,d\overline{X}. \quad (5)$$

For the case of dimensionless mass flow rate $\bar{j}_W$ = = const along the length, and boundary conditions $\mathrm{Re}_T^{**} = 0$ at $\overline{X} = 0$, we have

$$\mathrm{Re}_T^{**} = \mathrm{Re}_0\bar{j}_W(K+1)\overline{X}. \quad (6)$$

For the turbulent regime the heat transfer law on the impermeable surface has the form

$$\mathrm{St}_0 = 0.0126/\mathrm{Re}_T^{**0.25}\mathrm{Pr}^{0.75}. \quad (7)$$

The following boundary relation for a permeable surface has been obtained in [7]:

$$\Psi_s = \Psi_t(1 - b_T/b_{T\cdot cr})^2, \quad (8)$$

where

$$\Psi_t = \left(\frac{2}{\sqrt{\psi}+1}\right)^2. \quad (9)$$

Solving (6)–(9) simultaneously and using (2)–(4), we obtain

$$\left\{b_{T\cdot cr} - \frac{Kb_{T\cdot cr}^2}{2\Psi_t}\left[\sqrt{1 + \frac{4\Psi_t}{Kb_{T\cdot cr}}} - 1\right]\right\}(K+1)^{-0.25} = \frac{(\mathrm{Re}_0\overline{X})^{0.25}\mathrm{Pr}^{0.75}\bar{j}_W^{1.25}}{0.0126\tilde{U}}. \quad (10)$$

The left side of the last expression is some function F_1 depending on the temperature ratio

$$\psi = T_W/T_0, \quad d = T'/T_0,$$
$$K = (\psi - d)/(1 - \psi). \quad (11)$$

A calculation of function F_1 for the case of uniform injection of air was performed on an electronic computer for ten temperature conditions. The results are shown in Fig. 1, where curves 1–4 correspond to the regimes with $\psi < 1$, i.e., to conditions where cold gas is injected, this being typical for porous cooling; curves 5–10 correspond to $\psi > 1$, which are conditions often used in laboratory tests.

It may be seen from (11) that when $\psi = d$, the quantity $K = 0$ denotes critical injection, where the injected gas lifts the boundary layer from the wall and $T_W = T'$.

By removing the indeterminacy of function F_1 when $\psi = d$, we obtain that $F_1 = b_{T.cr}$ at this point, and the line joining such points for various values of d gives the critical injection limit.

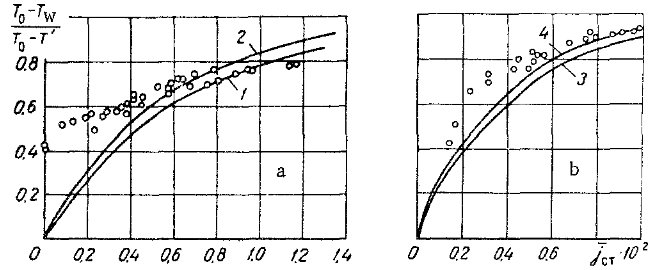

Fig. 2. Comparison of the calculation (curves) with test data (points) of Grootenhuis (a) and Bayley (b): 1 and 2) with d = 0.392, $Re_0 = 2\cdot 10^4$ and $5\cdot 10^4$; 3 and 4) d = 0.529, $Re = 1\cdot 10^5$ and $2\cdot 10^5$.

Use of the graph of Fig. 1 appreciably simplifies calculation of the temperature distribution of a porous wall. With the main flow parameters known and the velocity variation law $\tilde{U} = f(\overline{X})$, and for a given flow rate of injected gas, the right side of (10) is calculated: since it is equal to function F_1, ψ, and therefore T_W, are found from Fig. 1 for the specific value of d.

Figure 2 gives the calculated and experimental data for a plate with $\tilde{U} = 1$, the calculation being done for the mean section $\overline{X} = 0.5$ and for the extreme values of Reynolds number occurring in the tests.

It should be noted that for small values of dimensionless flow rate $\bar{j}_W < 0.4$, the test points of both authors lie above the theoretical relation. This is probably due to the fact that with decrease of temperature drop $T - T_W$ occurring for diminished gas injection rate, the effect of thermal leakage and other measurement errors begins to be appreciable.

NOTATION

U_0, U_1– velocity of oncoming stream and local velocity; $\tilde{U} = U_1/U_0$–dimensionless velocity; T_0, T_W, T'–temperatures of main stream, wall, and injected gas, respectively; $\overline{X} = x/L$ –dimensionless length; $\bar{j}_W$–dimensionless flow rate (ratio of specific flow rate of injected gas to that of main stream); $\Psi_S = (St/St_0)_{Re^{**}}$–relative change of St number at Re^{**} = idem; $St_0 = \alpha_0/c_p \gamma_0 U_0 \tilde{U}$–St number on impermeable surface, as defined by (7); $\psi = T_W/T_0$–temperature factor; $\Delta T = T_0 - T_W$–temperature head; Re_0, Re_T^{**}–Reynolds number values of main stream ($U_0 L/\nu_0$) and thermal boundary layer ($U_0 \delta_T^{**}/\nu_0$).

REFERENCES

1. P. Grootenhuis, Journal of the Royal Aeronautical Society, **63**, no. 578, 73–89, 1959.

2. W. H. Dorrance and F. S. Dore, JAS, 21, no. 6, 404–411, 1954.

3. H. S. Mickley et al., NACA, TN 3208, 1954.

4. F. J. Bayley, NGTE Report, R62, 1950.

5. P. N. Romanenko and V. N. Kharchenko, IFZh, no. 2, 1963.

6. S. S. Kutateladze and A. I. Leont'ev, The Turbulent Boundary Layer in a Compressible Gas [in Russian], Izd. SO AN SSSR, Novosibirsk, 1962.

7. Heat Transfer and Friction in the Turbulent Boundary Layer [in Russian], ed. S. S. Kutateladze, Izd. SO AN SSSR, Novosibirsk, 1964.

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